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# Non-linear waves in non-planar inhomogeneous dusty plasmas

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**Abstract.** Taking into account the inhomogeneity of the ion density and dust charging, the propagation of dust-ion acoustic solitary wave (DIASW) in a non-planar and inhomogeneous dusty plasmas is investigated analytically. The analytical expressions for the evolution of DIASW and the emitted radiation profiles of DIASW caused by the combined effects of non-planar geometry and inhomogeneity are obtained.

**PACS.** 52.25.Vy Impurities in plasmas – 52.35.Fp Electrostatic waves and oscillations (e.g., ion-acoustic waves)

Nonlinearity, inhomogeneity, dispersivity, and dissipativity are general features of the dusty plasmas. Hence, rich non-linear excitation of nonlinear coherent structures exists in dusty plasmas. Among them, dust ion-acoustic solitary waves (DIASW) and dust-acoustic solitary waves (DASW) would be the fundamental nonlinear coherent structures in dusty plasmas [1–5]. Experimental and theoretical investigations for nonlinear evolution of DIASW/DASW in dusty plasmas have been paid increasing interests. In recent years, the investigation of the DIASW/DASW are also extended to the nonplanar geometry [6–8] and result in considerable success in clarifying many aspects of the characteristics of DIASW/DASW. However, the investigations about DIASW/DASW in bounded nonplanar cylindrical/spherical geometry are all focused on homogeneous plasma in which the equilibrium quantities are taken as constants. In reality, the equilibrium states of dusty plasmas in space and laboratory are often nonuniform. In an inhomogeneous dusty plasma, the inhomogeneity of particle density and dust charging would play an important role and the properties of DIASW/DASW will be modified significantly [9–14]. When the soliton moves through the inhomogeneous non-planar dusty plasmas, soliton will experiences the instability arises both from the ring curvature (i.e., the non-planar geometry effect) and the inhomogeneity, and leads to the instability of sound emission from the soliton. However, the dynamics, the instability to sound emission and the profiles of the emitted sound waves of the DIAWSW/DASW remains an open question in dusty plasmas. It is the aim of the present paper to discuss these points analytically. Hence, in this letter, taking into account the ion density inhomogeneity and dust charging inhomogeneity, the propagation of DIASW in non-planar and inhomogeneous dusty plasmas is studied analytically. The analytical expressions for the evolution of DIASW and emitted radiation profiles are obtained. We expect that the present investigation would provide an insight understanding on the propagation of nonplanar DIASW in nonuniform dusty plasmas.

Here, we consider an unmagnetized collisionless plasma consisting of ions, electrons, and cold, extremely massive, microsized, negatively charged dust grains in non-planar cylindrical/spherical geometry. We assume that the DIASW propagates in the radial direction with cylindrical/spherical symmetry. In low temperature and pressure plasmas, the basic equations governing the dusty plasmas for this system are

$$\frac{\partial n_i}{\partial t} + \frac{1}{r^m}\frac{\partial (r^m n_i u_i)}{\partial r} = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial r} = -\frac{\partial \phi}{\partial r}, \tag{2}$$

$$\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m\frac{\partial \phi}{\partial r}\right) = Z_d n_d + n_e - n_i, \tag{3}$$

where $m = 0, 1, 2$ refer to one-dimensional, cylindrical, and spherical geometry, respectively. On ion-acoustic time scale, the electrons number density $n_e$ is given by the local thermodynamic equilibrium distribution

$$n_e = n_{e0}(r)\exp(\phi). \tag{4}$$

The equilibrium density of ions $n_{i0}(r)$ and electrons $n_{e0}(r)$ are inhomogeneous and vary with $r$ coordinate. The heavy dust grains in ion-acoustic time scale form a stationary background with a constant equilibrium density $n_d$. However, the grain charge $Z_d$ can subject to spatial variations

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as well as fluctuations. We assume the grain charge in equilibrium state ($Z_{d0}(r)$) is inhomogeneous. So, in the equilibrium state, the overall charge neutrality satisfy

$$n_{i0}(r) = n_{e0}(r) + Z_{d0}(r)n_d. \tag{5}$$

The dust charge variation $Z_d$ comes in through the charge current balance equation $-edZ_d/dt = I_e + I_i$, which is valid for grain charging arising from plasma currents due to electrons ($I_e$) and ions ($I_i$) reaching the grain surface. When the streaming velocities of the electrons and ions are much smaller than their respective thermal velocities, the expressions for the electron and ion currents for spherical grains of radius $a$ are given by $I_e = -\pi a^2 e(8T_e/\pi m_e)^{1/2} n_e \exp(-zZ_d)$, $I_i = \pi a^2 e(8T_i/\pi m_i)^{1/2} n_i(1 + zZ_d/\sigma_i)$, where $z = Z_{d0}(r_0)e^2/T_e a$, $\sigma_i = T_i/T_e$. It is well-known that the characteristic time for dust motion is of the order of tens of milliseconds for micrometer-sized grains, while the dust charging time is typically of the order of $10^{-8}$ s, therefore, the motion of dust is not so fast that the contribution from the electron current to dust is balance for the ions. It follows that $dQ_d/dt \ll I_e, I_i$ and the current balance equation reads $I_e + I_i = 0$ and we obtain

$$\alpha n_{e0}\exp(\phi - zZ_d) - n_i\left(1 + \frac{z}{\sigma_i}Z_d\right) = 0, \tag{6}$$

where $\alpha = \sqrt{\mu_i/\sigma_i}$, $\mu_i = m_i/m_e$. In the above equations, the density of electrons and ions are normalized by $n_{i0}(r_0)$. The variables $t$, $r$, $n_d$, $u_i$, and $\phi$ are normalized by $\omega_{pi}^{-1}(r_0) = (m_i/4\pi n_{i0}(r_0)e^2)^{1/2}$, $\lambda_D = (T_e/4\pi n_{i0}(r_0)e^2)^{1/2}$, $n_{i0}(r_0)Z_{d0}(r_0)$, $C_s = (T_e/m_i)^{1/2}$, and $T_e/e$, respectively. Equations (1–6) govern the propagation of DIASW in inhomogeneous non-planar dusty plasmas.

To obtain the nonlinear evolution of the DIASW, we define two slow variables

$$R = \epsilon^{3/2}r, \quad \xi = -\epsilon^{1/2}\left[t + \int \frac{1}{v_0(r')}dr'\right], \tag{7}$$

where $\epsilon$ is a small ordering parameter and $v_0(r)$ is the velocity of the moving frame to be determined later. The dependent variables can be scaled as

$$n_i = n_{i0}(R) + \epsilon n_1(R,\xi) + \epsilon^2 n_2(R,\xi) + \cdots, \tag{8}$$

$$\phi = \epsilon\phi_1(R,\xi) + \epsilon^2\phi_2(R,\xi) + \cdots, \tag{9}$$

$$u = \epsilon u_1(R,\xi) + \epsilon^2 u_2(R,\xi) + \cdots, \tag{10}$$

$$Z_d = Z_{d0}(R) + \epsilon Z_{d1}(R,\xi) + \epsilon^2 Z_{d2}(R,\xi) + \cdots, \tag{11}$$

where the equilibrium quantities are time independent and are slowly variable functions of $R$ only. Then, substituting the above expansions into equations (1–6) and collecting the terms in the different powers of $\epsilon$, we can obtain each $n$th-order reduced equation. To the leading order, we have

$$n_1 = \frac{n_{i0}}{v_0^2}\phi_1, \quad u_1 = -\frac{1}{v_0^2}\phi_1 \tag{12}$$

$$Z_{d1} = \frac{1-\delta}{\beta + (1-\delta)/Z_{d0}}\phi_1, \tag{13}$$

where $\delta = n_{e0}/n_{i0}$, $\beta = z[1 + 1/(\sigma_i + zZ_{d0})]$, $v_0^2 = [\beta + (1-\delta)/Z_{d0}]/[\beta\delta + (1-\delta)/Z_{d0}]$. At the next order, we have

$$\frac{\partial n_2}{\partial\xi} + \frac{1}{v_0}\frac{\partial}{\partial\xi}(n_{i0}u_2 + n_1 u_1) - \frac{\partial}{\partial R}(n_{i0}u_1) - \frac{m}{R}n_{i0}u_1 = 0, \tag{14}$$

$$\frac{\partial u_2}{\partial\xi} + \frac{1}{v_0}u_1\frac{\partial u_1}{\partial\xi} + \frac{1}{v_0}\frac{\partial\phi_2}{\partial\xi} - \frac{\partial\phi_1}{\partial R} = 0, \tag{15}$$

$$n_{i0}\left(\phi_2 - zZ_{d2} + \phi_1^2/2 - z\phi_1 Z_{d1} + z^2 Z_{d1}^2\right) = (\beta - z)(n_{i0}Z_{d2} + n_{i1}Z_{d1}) + n_{i2}. \tag{16}$$

The solvability condition for equations (14–16) reads

$$\frac{\partial\phi_1}{\partial R} + A\phi_1\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} + \frac{1}{2}\left[\frac{m}{R} + \frac{d}{dR}\ln\left(\frac{n_{i0}}{v_0}\right)\right]\phi_1 = 0, \tag{17}$$

where

$$\begin{aligned} A(R) = &-\frac{3}{2v_0^3} + \frac{1}{2v_0} \\ &- \frac{zZ_{d0}}{v_0}\frac{(1-\delta)^2}{(1-\delta+\delta\beta Z_{d0})(1-\delta+\beta Z_{d0})} \\ &\times\left[1 - \frac{zZ_{d0}(1-\delta)}{1-\delta+\beta Z_{d0}} + \frac{1}{v_0^2(\sigma_i + zZ_{d0})}\right], \\ B(R) = &-\frac{\beta Z_{d0}}{2v_0^3(1-\delta+\beta\delta Z_{d0})n_{i0}}. \end{aligned}$$

Equation (17) is a variable coefficients KdV equation describing the small-amplitude DIASW in the radially inhomogeneous non-planar dusty plasmas. The last term in equation (17) refers to the combined effect of non-planar geometry and inhomogeneity. In the homogeneous case, equation (17) reduces to the cylindrical/spherical KdV equation describing the small-amplitude DIASW in the radially homogeneous dusty plasmas.

Introducing the transformations $\phi_1 = -(6B/A)u$, $\chi = \int BdR$, equation (17) reduces to

$$\frac{\partial u}{\partial\chi} - 6u\frac{\partial u}{\partial\xi} + \frac{\partial^3 u}{\partial\xi^3} = P(\chi)u, \tag{18}$$

where

$$P(\chi) = -\frac{1}{2}\frac{d}{d\chi}\ln\left[\frac{n_{i0}}{v_0}\left(\frac{B}{A}\right)^2\right] - \frac{m}{2BR}.$$

For a planar and homogeneous plasma, i.e., $P(\chi) = 0$, equation (18) is a one-dimensional KdV equation, which has a single-soliton solution of the form

$$u = -2\kappa^2\mathrm{sech}^2 Z, \quad Z = \kappa[\xi - \vartheta(\chi)], \tag{19}$$

where $\vartheta(\chi) = 4\kappa^2\chi + \vartheta_0$ is the soliton center (with $d\vartheta/d\chi = 4\kappa^2$ being the soliton velocity in the $\xi - \chi$

reference frame), while $\kappa$ and $\vartheta_0$ are arbitrary constants presenting the soliton's amplitude and initial position, respectively.

Now we discuss the evolution of DIASW of equation (18) in inhomogeneous non-planar dusty plasmas. An analytical solution of equation (18) can be obtained by using a suitable perturbation theory for soliton. In regions of weak inhomogeneous and for larger soliton ring radius (this is the most interesting case), the right hand side of equation (18) can be treated as a small perturbation. The solution of equation (18) can be expressed as [15]

$$u = u_s + u_r \tag{20}$$

where $u_s$ is the soliton part, which has the same functional form as in the unperturbed homogeneous case (cf., Eq. (19)), but with the soliton parameters $\kappa$ and $\vartheta$ being now unknown functions of $\chi$. The contribution $u_r$,being of the same order of the smallness as $P(\chi)$, denotes the radiation part of the solution due to the effect of the inhomogeneity. According to [15], the soliton's amplitude $\kappa(\chi)$ and center $\vartheta(\chi)$ are determined by

$$\frac{d\kappa}{d\chi} = \frac{2}{3}\kappa P(\chi), \quad \frac{d\vartheta}{d\chi} = 4\kappa^2 + \frac{1}{3\kappa}P(\chi). \tag{21}$$

Integrating of equation (21), one can obtain the soliton's amplitude $\kappa^2(R)$

$$\kappa^2(R) = \kappa^2(R_0)\left(\frac{R}{R_0}\right)^{-2m/3}\left[\frac{n_{i0}(R)}{n_{i0}(R_0)}\right]^{-2/3}\left[\frac{v_0(R)}{v_0(R_0)}\right]^{2/3} \times\left[\frac{A(R)}{A(R_0)}\right]^{4/3}\left[\frac{B(R)}{B(R_0)}\right]^{-4/3}, \tag{22}$$

and the center $\vartheta(\chi)$ expressed in terms of the slow variable $R$

$$\vartheta(R) = \int 4\kappa^2(R)B(R)dR - \int \frac{1}{6\kappa(R)} d\ln\left\{R^m \frac{n_{i0}}{v_0}\left[\frac{B(R)}{A(R)}\right]^2\right\} \tag{23}$$

where $\kappa(R_0)$ is the soliton amplitude at $R = R_0$. It is clear that, because of the ring curvature (nonplanar geometry) and the inhomogeneous of the background, the DIASW amplitude is modified significantly. In a inhomogeneous non-planar dusty plasmas, the amplitude of DIASW vary according to equation (22). Following [15], for $|Z| \gg 1$, the radiation part of the soliton is expressed by

$$u_r = \frac{1}{12\kappa(R)B(R)} \times\left[\frac{m}{2R} + \frac{d}{dR}\ln\left(\frac{n_{i0}}{v_0}\left|\frac{B(R)}{A(R)}\right|\right)\right](1 - \tanh Z). \tag{24}$$

One can find from equation (24) clearly that the sound emission from soliton is caused by the combined effects of the ring curvature (nonplanar geometry) and the inhomogeneous of the background.

In summary, the evolution of non-planar DIASW in a inhomogenenous dusty plasmas is studied by a perturbation method. Theoretical analysis shows that the DIASW is governed by a variable-coefficients KdV equation. The reduction to the KdV equation may be useful to understand the dynamics of DIASW and will help to get a deeper insight into the physics of the DIASW in inhomogeneous non-planar dusty plasmas. The analytical expressions for the evolution of soliton and emitted radiation profiles are also obtained.

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